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On the Heat Death and the Big Crunch

In Memory of Jerry Ericksen

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Abstract

This article is an exercise in elementary mechanics and thermodynamics. It emphasizes the basic difference between mechanics and thermodynamics which lies in the irreversibility of the latter theory. Clausius [1] (Poggendorff's Annalen der Physik, p. 125, 1865) – the discoverer of entropy – was led to express the dichotomy in the famous slogan

die Energie der Welt ist constant, and
die Entropie der Welt strebt einem Maximum zu.

(Gibbs put these slogans on top of his comprehensive memoir [2] (Gibbs, On the Equilibrium of Heterogeneous Substances, pp. 108–248, 1876).)
And again Clausius [3] (Über den zweiten Hauptsatz der mechanischen Wärmetheorie, A lecture given at the general session of German natural scientists and physicians in Frankfurt/M on September 23, 1867, 1867) invented the notion of the heat death as the final destiny of the world. Nowadays it has become quiet around the heat death. Says Asimov [4] (Asimov's Biographical Encyclopedia of Science and Technology, 1975): "Though the laws of thermodynamics stand as firmly as ever, cosmologists ... [show] a certain willingness to suspend judgement on the matter of heat death." And we, the authors of this article, might add that present-day cosmologists know so little about the universe that heat death is a tiny spot in the sea of ignorance and therefore not worthy of discussion by serious scientists. All the more reason to take a fresh look at the subject.

Now, however, this article is most definitely not a contribution to formal and technical cosmology as described in the monograph [5] (Cosmology, 2008) by S. Weinberg, or the popular booklet [6] (Fundamentals. Ten Keys to Reality, 2021) by F. Wilczek. Far from it! We treat a monatomic ideal gas under adiabatic conditions in two situations: The gas in a cylinder under a locked piston which – at some instance – is released; and a homogeneous cloud of gas at rest when – at some time – gravitation is "switched on".

It is only by a big leap of imagination that the two cases may be viewed as models for the universe. We also admit that we do not know whether the universe is adiabatic and free of working. Clausius and Gibbs thought so and we follow authority and investigate the validity of the prediction of the heat death and the related idea of the big crunch under these assumptions. We shall confirm the heat death as a very hot event but refute the big crunch.

Keywords Irreversibility · Big bang · Big crunch

Mathematics Subject Classification (2010) 80 · 85

Extended author information available on the last page of the article

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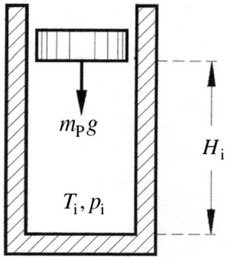

Fig. 1 A gas under a piston

1 Cylinder-Piston Problem

Fig. 1 shows a monatomic ideal gas with temperature T_i in a cylinder enclosed by a piston of weight $m_P g$ which is locked in position at the height H_i. The pressure in the gas is $p_i = \frac{m_G (R/M_r) T_i}{H_i A}$ and the internal energy is $E^i_{\text{int}} = z R/M_r T_i m_G$, where m_G is the mass of the gas and A is the cross-section of the cylinder. z is equal to $3/2$ for a monatomic ideal gas, R is the ideal gas constant and M_r is the relative atomic mass of the gas, relative to $1/12$ of the C^{12} atom by international agreement. We assume that the gas is adiabatically isolated, not only by the cylinder walls but also against the piston. The outside pressure is zero. Also the weight of the gas is neglected. When the piston is released, it will move in general and the gas will suffer an irreversible, highly turbulent motion until equilibrium is reached. During the process the energy of gas and piston is conserved and, when equilibrium is reached, the final height H_f and the final temperature T_f are related by

$$\text{the balance of forces on the piston} \quad \frac{H_f}{T_f} = \frac{m_G R/M_r}{m_P g}$$
$$\text{and conservation of energy} \quad z m_G R/M_r T_f + m_P g H_f = z m_G R/M_r T_i + m_P g H_i. \tag{1}$$

These are two equations for H_f and T_f so that we obtain

$$H_f = \frac{1}{1+z}\left(H_i + \frac{m_G}{m_P}\frac{z R/M_r}{g} T_i\right) \text{ and } T_f = \frac{1}{1+z}\left(z T_i + \frac{m_P}{m_G}\frac{g}{R/M_r} H_i\right). \tag{2}$$

Thus H_f may both be smaller and bigger than H_i depending on the mass of the piston and the same is true for T_f in relation to T_i. It is remarkable, perhaps, that for a very heavy piston – ultimately for an infinitely heavy one – we have

$$H_f \underbrace{\rightarrow}_{\text{for } m_P \to \infty} \frac{1}{1+z} H_i \quad \text{and} \quad T_f \underbrace{\rightarrow}_{\text{for } m_P \to \infty} \infty, \tag{3}$$

so that the height of the piston undergoes a moderate change, while the temperature grows unlimited. It is the pressure in the gas that keeps the heavy piston balanced; and the pressure is high because the temperature is high and not because the volume of the gas is small.

It may be worth mentioning that in an adiabatic *reversible* process, i.e. where the motion of the piston is controlled to be slow, the so-called "adiabatic equation of state", requires $H^{1/z} T = H_i^{1/z} T_i$ so that for $T_f \to \infty$ we have $H_f \to 0$. Thus in this example we see the difference between irreversibility and reversibility drastically displayed.

Thermodynamics is wonderful. It serves both the mechanic and the cosmologist. And while the mechanic sees in Fig. 1 a primitive shock absorber, the cosmologist sees a model for the universe, perhaps; a poor model to be sure for both of them. The cosmologist will be reminded of the heat death but he will still be disappointed with the model, if he puts his trust in the idea of the "big crunch", the hypothetical counter part of the hypothetical big bang, when the universe comes crushing down at the end. Indeed, the gas in the cylinder is not crushed to any significant degree under even a very large weight.

While Clausius says nothing about the temperature of the heat death, – calling it a "dead stagnant state" –, most people seem to think that the heat death is a hot event, because they associate the expected eventual collapse of the universe with an unlimited increase of temperature.[1] As was indicated before we – as well – use the expression to describe an infinitely hot state; well aware that others think of the heat death as a very cold state, and still others are non-committal. It does not really matter, because we calculate temperatures and volumes and it is only incidental whether they coincide with anybody else's concept of heat death and big crunch.

We are, however, not quite finished with the cylinder-piston gadget: It is also interesting to consider the extreme of a very light piston. In that case we have

$$H_f \underbrace{\rightarrow}_{\text{for } m_P \rightarrow 0} \infty \quad \text{and} \quad T_f \underbrace{\rightarrow}_{\text{for } m_P \rightarrow 0} \frac{z}{1+z} T_i . \tag{4}$$

It is obvious intuitively that the gas in that case pushes the – essentially non-existing – piston far out. And maybe it is not as clear to the non-thermodynamicist that the temperature stays finite. It drops, but not very far.

This latter case, of course, has not even remote relevance to the collapsing universe. If anything it might give us ideas about the expanding universe. One of those ideas is that the temperature at the end of the expansion is not very different from the initial one.

2 Gas Sphere

2.1 The Problem

2.1.1 Turbulent Motion and Final Equilibrium

Let us investigate a case which is a little more like the universe than the cylinder-piston case. We envisage a monatomic ideal gas at rest and homogeneously distributed with density ρ_i in a sphere with radius R_i and with the temperature T_i. The sphere is enclosed by a rigid spherical roof.[2] Its mass is $M = \frac{4\pi}{3}\rho_i R_i^3$ and the internal energy is $E_{\text{int}}^i = z^R/_{M_r} T_i M$. And now let gravitation be "switched on".

Afterwards the gas inside the sphere has a potential energy due to the pairwise interaction of any two mass elements dm and dm' at points P and Q which reads

$$dE_{\text{pot}} = -G\frac{dm \cdot dm'}{r_{PQ}},$$

[1] Clausius – and most people at his time – thought of the universe as static, so that ideas of expansion and collapse did not enter their minds. And, of course, the big crunch is a more modern concept.

[2] It is true that this model has a somewhat medieval – or even ancient – touch. But then, it is no more strange than the image of the universe as the three-dimensional surface of a four-dimensional sphere in space-time. And, after all, this is an exercise in classical mechanics and thermodynamics.

where G is the gravitational constant and r_{PQ} is the distance of the points P and Q. The total potential energy assumes the form

$$E_{\text{pot}} = -G\int_0^{R_i}\int_0^{\pi}\int_0^{2\pi}\int_0^{R_i}\int_0^{\pi}\int_0^{2\pi}\frac{\rho(r,\vartheta,\phi)r^2\sin\vartheta\, dr d\vartheta d\phi\, \rho(r',\vartheta',\phi')r'^2\sin\vartheta' dr' d\vartheta' d\phi'}{\sqrt{r^2+r'^2-2rr'(\cos\vartheta\cos\vartheta'+\sin\vartheta\sin\vartheta'\cos(\phi-\phi'))}}, \tag{5}$$

where r, ϑ, ϕ and r', ϑ', ϕ' are the spherical coordinates of dm and dm' in relation to the center of the sphere.

In the gravitational field the gas begins to adjust to the new situation in an irreversible adiabatic process; or that is what we assume: adiabaticity. And the energy is constant, because the "roof" does not move. The adjustment is highly turbulent, but eventually – after what may be a long time – it comes to an end. According to Clausius and Gibbs that end is characterized by a maximum of entropy S reached by a process of constant energy $E = E_{\text{pot}} + E^i_{\text{int}} + E_{\text{kin}}$. Or in mathematical language the end is characterized by

$$\delta(S - \lambda E) = 0, \tag{6}$$

where λ is a Lagrange multiplier taking care of the constraint of constant energy, and δ is an arbitrary variation of the process $T(x_n,t)$, $v_i(x_n,t)$, $\rho(x_n,t)$. v_i is the velocity of the gas at the point x_n and time t. The Gibbs equation for the differential of the specific entropy s

$$ds = \frac{1}{T}\left(z^R/_{M_r}dT + pd\frac{1}{\rho}\right) \tag{7}$$

– result of the first and second law of thermodynamics – implies necessary conditions for the validity of the variational principle (6); namely isotropy about the center of the sphere and final fields T^f, v_i^f, and p^f satisfying

$$T^f(x_n,t) = \text{const},\ v_i^f(x_n,t) = 0,\ \text{and}\ p_f(r) = p_f(0) - \frac{G}{4\pi}\int_0^{M(r)}\frac{M(\alpha)}{\alpha^4}dM(a), \tag{8}$$

where $M(r)$ is the mass within the sphere of radius r.

Thus when equilibrium is reached – at the end of the adjustment – the gas has come to rest with a homogeneous temperature T_f and – generally – a high-density-kernel of radius R_f. That is the situation we look at. The potential energy in this situation is

$$E^f_{\text{pot}} = -16\pi^2 G\int_0^{R_i}\int_0^{r}\rho_f(r,t)\rho_f(\alpha,t)\alpha^2 r\, d\alpha\, dr. \tag{9}$$

2.1.2 Mass Density of an Ideal Gas in a Gravitational Field

According to Newton's law of gravitation, – which is equivalent to $(8)_3$ – the gas pressure is given by the differential equation in the eventual equilibrium

$$\frac{dp_f}{dr} = -\rho_f G\frac{M(r)}{r^2}\ ,\ \text{where}\quad M(r) = \int_0^r \rho_f 4\pi r^2 dr. \tag{10}$$

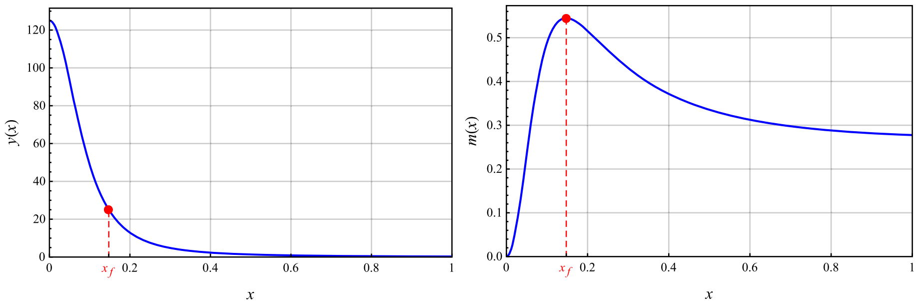

Fig. 2 The densities $\rho_f(r)$ and $m(r) = \rho_f(r)4\pi\, r^2$ for $y(0) = 125$, and $B = 6.329$. The definition of R_f or x_f

With the thermal equation of state of an ideal gas this leads to a differential equation for the density ρ_f, viz.

$$\frac{d^2\rho_f}{dr^2} - \frac{1}{\rho_f}\left(\frac{d\rho_f}{dr}\right)^2 + \frac{2}{r}\frac{d\rho_f}{dr} = -\frac{4\pi G}{{}^{R}/_{M_r}T_f}{\rho_f}^2. \tag{11}$$

For a numerical solution we need to introduce dimensionless variables

$$y = \frac{\rho_f}{\rho_i}, \quad x = \frac{r}{R_i} \quad \text{and} \quad B = \frac{3GM}{{}^{R}/_{M_r}T_f}\frac{1}{R_i} = 3c\frac{T_i}{T_f} \tag{12}$$

where $c = \frac{{}^{GM}/_{R_i}}{{}^{R}/_{M_r}T_i}$ is a characteristic dimensionless parameter representing the ratio of the gravitational and the internal energy for initial values of temperature and radius. In dimensionless variables the differential equation assumes the form

$$y'' - \frac{1}{y}(y')^2 + \frac{2}{x}y' + By^2 = 0. \tag{13}$$

Of course the outer radius is still R_i. But inside the volume of the "roofed" sphere a structure has formed with a large density in the center and rapidly decaying density away from the center. This is dictated by the differential equation for boundary values $\rho_f(0)$, and $\rho'_f(0) = 0$. Fig. 2 shows a typical mass density distribution $\rho_f(r)$ in $0 < r < R_i$, – with a high density part which we may call the "kernel" –, and the mass within a thin shell of thickness dr, $m(r)dr = \rho_f 4\pi r^2 dr$.

To fix the ideas let us say that an outside observer of the final equilibrium will see a dense kernel of radius R_f and a thinly populated exterior of that kernel for $R_f < r < R_i$. However, we also see that the space between R_f and R_i is not swept clean of matter by the formation of the kernel. A close look at the solution of the differential equation reveals that the density $\rho_f(r)$ inside the shell $R_f < r < R_i$ drops approximately inversely proportional to r^2. Yet, although its density is small, the mass in the shell cannot be neglected, because the increase of the volume of the shell offsets the decrease of the density; such that each shell of thickness dr within $R_f < r < R_i$ contains approximately the same mass.

Obviously there is a certain arbitrariness about the definition of R_f, the radius of the kernel. A natural choice seems to be that R_f should be defined as the abscissa of the maximum of $m(r)$. We choose it so, see Fig. 2.

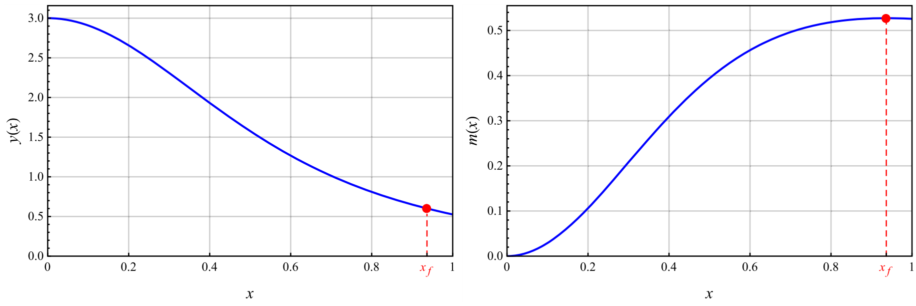

Fig. 3 $y(x)$ and $m(x)$ for $y(0) = 3$, $B = 6.23$. Large value $x_f \lessapprox 1$

2.1.3 The Mass Constraint

The total mass of the final distribution must be equal to the initial one, so that a relevant solution must satisfy the constraint

$$\int_0^{Rf} \rho_f(r) 4\pi r^2 dr + \int_{R_f}^{R_i} \rho_f(r) 4\pi r^2 dr = \frac{4\pi}{3} \rho_i {R_i}^3 \quad \text{or in dimensionless form}$$

$$\int_0^{x_f} y(x) x^2 dx + \int_{x_f}^{1} y(x) x^2 dx = \frac{1}{3} \tag{14}$$

We shall refer to this constraint as the *mass constraint*. To satisfy it requires a cumbersome method of shooting for the right combination of $y(0)$ and B; the shooting parameter is B. Combinations of $y(0)$ and B which satisfy the mass constraint and the corresponding x_f are shown in Table 1.

The lines starting with $y(0) = 125$ and $y(0) = 3$ correspond to the graphs of Figs. 2 and 3, respectively. They represent two instructive cases: In Fig. 2 we observe a well-developed kernel, while in Fig. 3 the kernel is just beginning to form. We may say that in Fig. 2 the kernel of the density distribution is well concentrated in the center of the sphere while in Fig. 3 it spreads over the whole sphere.

And while x_f increases monotonically with decreasing values of $y(0)$, B is non-monotonic; the crosses in the table mark minima and maxima. Fig. 4 represents the graph of the function[3] $\frac{0.5}{B(x_f)}$ by the upper blue dotted line – taken from the values of Table 1. We have

$$\frac{T_f}{T_i} = 6c \frac{0.5}{B({}^{R_f}/_{R_i})}, \tag{15}$$

which is equivalent to (12). This is the relation between T_f and R_f that is enforced by the mass constraint.

We shall now proceed to calculate T_f, – the final temperature –, and R_f, – the radius of the kernel in the sphere –, separately. For this we need the equation expressing energy conservation. The purpose is to find out whether those final values correspond in any way to what we imagine the temperature of the heat death to be, and whether R_f is such that we may properly call the adjustment to gravitation a big crunch.

[3] We choose to represent $0.5/B(x)$, – rather than $B(x)$ itself –, because this is the dimensionless form of the internal energy in the final state, as will soon become clear.

Table 1 Values for $y(0)$, B, R_f, and $\frac{E_{\text{pot}}}{9GM^2/R_i}$

$y(0) = \frac{\rho_f(0)}{\rho_i}$	$B = 3\frac{GM}{R/M_r}\frac{1}{T_f R_i}$	$x_f = \frac{R_f}{R_i}$	$-e = \frac{-E_{\text{pot}}}{9GM^2/R_i}$
1.00E+10	6.0085	0.00002	0.1110
5.00E+09	6.0119	0.00002	0.1110
2.00E+09	6.0132	0.00004	0.1109
1.00E+10	6.0102	0.00005	0.1108
5.00E+05	6.1135	0.0023	0.1100
3.00E+05	6.1355	0.0030	0.1093
2.00E+05	6.1435	0.0036	0.1088
1.00E+05	6.1294	0.0052	0.1083
40000.00	6.0412	0.0083	0.1087
30000.00	5.9963	0.0097	0.1091
20000.00	5.9211	0.0119	0.1100
10000.00	5.7731	0.0168	0.1122
4000.00	5.5955	0.0275	0.1163
3000.00	5.5583	0.0314	0.1178
2000.00	5.5303×	0.0389	0.1200
1000.00	5.5631	0.0541	0.1226
800.00	5.5978	0.0613	0.1231
600.00	5.6603	0.0700	0.1232×
400.00	5.7824	0.0844	0.1227
125.00	6.3290	0.1436	0.1158
80.00	6.5962	0.176	0.1115
50.00	6.8910	0.2214	0.1064
30.00	7.1982	0.278	0.1004
15.00	7.5037	0.3799	0.0922
12.50	7.5416	0.4181	0.0901
10.00	7.5503×	0.4673	0.0875
7.50	7.4815	0.5446	0.0843
5.00	7.1699	0.6875	0.0800
4.00	6.8513	0.7807	0.0778
3.00	6.2306	0.9311	0.0751
2.80	6.0392	0.9825	0.0745

2.2 Energy Conservation

Energy conservation during the adjustment of the gas under gravitation reads

$$E^f_{\text{pot}} + z\,{}^R\!/_{M_r} T_f M = E^i_{\text{pot}} + z\,{}^R\!/_{M_r} T_i M \quad \text{or} \quad e(x_f) + \frac{0.5}{B(x_f)} = \frac{1}{6}\frac{1}{c}, \tag{16}$$

since $E^i_{\text{pot}} = 0$, because gravitation is absent in the initial state. Note also that the kinetic energy vanishes at the initial and final states. e is the dimensionless form of E^f_{pot} defined on top of Table 1. z was set equal to $3/2$ in $(16)_2$ as is appropriate for a monatomic gas.

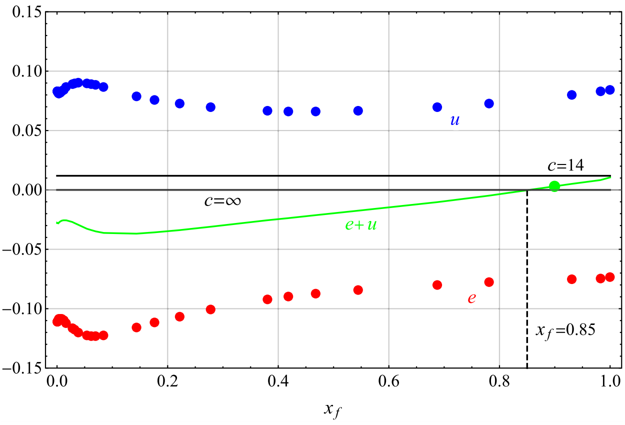

Fig. 4 E^f_{pot}, $\frac{3}{2}{}^R/_{M_r} T_f M$, and $E^f_{\text{pot}} + \frac{3}{2}{}^R/_{M_r} T_f M$, or $e(x_f)$, $u(x_f)$, and $e(x_f) + u(x_f)$

Fig. 4 shows the left hand side of (16) by the green graph. Note that $u(x_f) = \frac{0.5}{B(x_f)}$ is the dimensionless form of the internal energy in the final state; it is represented in Fig. 4 by the upper blue dotted line.

The desired solution R_f may be read off as the abscissa of the point of intersection of the graph with the horizontal line $\frac{1}{6c}$ which is the initial internal – and total – energy in dimensionless form. For instance for $c = \frac{GM/R_i}{R/_{M_r} T_i} \approx 40$ the intersection occurs for $x_f = 0.9$ (the green dot in Fig. 4) which means that the kernel has 90% of the radius of R_i. Once this is known, we can calculate $\frac{T_f}{T_i}$ from (15) and the table and obtain $T_f \approx 19\,T_i$ so that the temperature has increased during the irreversible contraction due to gravitation.

Thus our problem can now be solved for any value of M – and T_i – and it remains to discuss the result for interesting cases.

3 Discussion

Let c be our input variable; it may vary between zero and infinity depending on the mass M. For this discussion we ignore the influence of T_i assuming that it has some finite value.

Considering that it was not easy for us to calculate the graphs of E^f_{pot} and $E^f_{\text{int}} = z^R/_{M_r} T_f M$ for small values of x_f, it is ironic that we do not need those parts for the discussion of results. Indeed, the interesting part of the graph $E^f_{\text{pot}} + E^f_{\text{int}}$ occurs only where this function and the table are positive valued, i.e. for $x_f > 0.85$.

Also for $0 < c < 14$ corresponding to very small and intermediate masses $0 < M < 14\frac{R/_{M_r} T_i}{G/R_i}$ there is no point of intersection between $E^f_{\text{pot}} + E^f_{\text{int}}$ – the solid graph of Fig. 4 – and the horizontal line $\frac{1}{6 \cdot c}$, because the mass is too small to settle into an identifiable kernel under gravitation. This range is of little or no interest for us, since nothing drastic can possibly occur: The final central density is smaller than $\rho_f(0) \approx 2.8\rho_i$ and the final temperature is 7 times bigger than T_i. That is too little to speak of a heat death as an infinitely hot event and certainly no cataclysmic big crunch occurs here, – for small and moderate values of M.

When M is bigger, we may reach the intermediate value of c, e.g. $c = 40$, that was discussed before. A kernel is formed with $R_f \approx 0.9R_i$, and the temperature T_f grows beyond T_i but it remains moderate, – below $19\,T_i$.

Obviously the final situation becomes hotter as c tends to infinity for a very large total mass M. This happens for $R_f \approx 0.85\, R_i$ and the temperature T_f is infinite, – along with c. Surely that means heat death! But still no big crunch occurs!

Indeed, the ultimate value $R_f \approx 0.85 \cdot R_i$ means that the final kernel fills about 39% of the sphere. This is roughly the same moderate degree of "crunching" which is suffered by the gas in Section 1 under the infinitely heavy cylinder. Therefore while our model suffers a heat death, it does not undergo a big crunch even under the gravitational collapse of a very big mass.

The gas will suffer a damped oscillation until dissipation kills off all kinetic energy. This may take a long time but in the end it settles in a finite volume and – for infinite mass – with infinite temperature.

We do not tire to repeat that this is an exercise in elementary mechanics and thermodynamics of an ideal gas. It is clear that the hot gas with $T = \infty$ will not remain a monatomic ideal gas, even if the sphere ever contained one. Wilczek envisages a turbid soup of quarks and gluons intransparent for photons, and it is doubtful, if those obey the thermal equation of state of an ideal gas.[4]

For arbitrary constitutive equations our results will be quantitatively different, but we believe that qualitatively they will be the same; at least as long as the thermodynamic stability conditions are observed: positive specific heats and positive compressibility, etc. Those are trivially observed by ideal gases while for other bodies they follow from the second law of thermodynamics.

Really nobody in cosmology knows anything. Modern cosmology is a free-for-all field with ideas that are truly fantastic, more so – much more so – than those exploited in this paper. Actually one of the authors (IM) has attempted to contribute to the field in order to refute the modern idea that the universe be expanding in an accelerated manner. But that contribution fell into a black hole. See [8].

In the end, if pressed hard, cosmologists tend to fob you off with quotations from the writings of Saint Augustin who – nearly two thousand years ago – speculated about God's role before He set the universe in motion. Saint Augustin did not know! But at least he will know now, because he had time to ask Him.

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Declarations

Competing interests The authors declare no competing interests.

[4] Also, a hot gas in equilibrium has large fluctuations but Clausius did not consider those. It was Boltzmann [7] who speculated on fluctuations in his mind – expanding cosmology and interpretation of time. But nothing came of those either.

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