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A benchmark proposal

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Decision making in structural engineering problems under polymorphic uncertainty - A benchmark proposal

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Abstract

The treatment of diverse uncertainties is an important challenge in structural engineering problems, especially from the viewpoint of realistic analysis. Inaccuracy and variability are always present and have to be quantified by either probabilistic, possibilistic, polymorphic or other uncertainty approaches. Regardless to the applied uncertainty quantification method, the numerical predictions have to be useful for structural assessment and decision making. The authors propose in this contribution a benchmark example of a portal frame structure including various uncertainties. The goal of this benchmark study is to compare justifications and decisions provided by probabilistic and non-probabilistic methods with respect to clear challenges of decision making with and without measurements and data assimilation. The engineering problem itself is simple enough to understand and complex enough not to be reduced to a simple formula with uncertain parameters.

Keywords *benchmark; polymorphic uncertainties; portal frame; decision making; data assimilation*

1 Introduction and goals

The treatment of diverse uncertainties is an important challenge in structural engineering problems, especially from the viewpoint of realistic analysis. Inaccuracy and variability are always present and have to be quantified by either probabilistic, possibilistic, polymorphic or other uncertainty approaches. Regardless to the applied uncertainty quantification method, the numerical predictions have to be useful for structural assessment and decision making. Dealing with uncertainties is since long subject of research in several fields like weather and climate predictions [17], geosciences [8] or nuclear energy [4]. In structural engineering, uncertainty quantification is mainly related to the design and optimization problems [6, 9, 19, 20, 10, 16], geotechnics [11] or system and parameter identification [18].

In spite of numerous publications, there were only very few comparative studies involving epistemic and aleatory uncertainties simultaneously, i.e. dealing with polymorphic uncertainties [5]. Benchmark studies on special problems usually help comparing diverse methods and approaches with respect to their efficiency and accuracy and developing a standard. Some relevant benchmarks are known in nuclear energy [2], geosciences [1], control problems [13] or structural health monitoring [18]. However, benchmarks dealing with polymorphic uncertainties in structural engineering are very rare. The recent test bed example [14] originates from a joint activity of the Priority Programme SPP 1886 of the German Research Foundation (DFG) entitled "Polymorphic uncertainty modelling for the numerical design of structures". The goal is an objective comparison of various non-probabilistic methods with respect to decision making in engineering problems with polymorphic uncertainties.

The present contribution proposes a benchmark of a structural engineering problem that pursues several typical goals: (i) encourage researchers to develop new methods for polymorphic uncertainty modeling, (ii) provide a problem description and a measurement data set to test such new methods under realistic conditions, and (iii) allow an objective comparison of proposed methods based on a true data set of input and output. At that, the authors realize the need for a benchmark that would consider interpretable uncertainty for decision making based on the polymorphic uncertainty.

The considered engineering system is a portal frame under vertical and horizontal point loads. It is a simple mechanical system that helps avoiding false interpretation of results with respect to system behavior and failure states. On the other hand, it possesses two competing failure modes, material failure and stability failure, which make the limit state function strongly nonlinear and sensitive to uncertainties. Such a combination of failure modes is typical for many structural systems. The benchmark should demonstrate how real decision making can work in the case of polymorphic uncertainties. One of the crucial issues is an objective comparison of results with different origin, namely, probabilistic and non-probabilistic ones. Perhaps, this could be done on the basis of the decisions made. The second challenge of the benchmark deals with the question how data assimilation can help in the decision making. Finally, a design problem under polymorphic uncertainty is given in which more demanding operation requirements have to be fulfilled.

2 Computational model

2.1 Reference structure: portal frame

A portal frame structure consisting of two columns of height H and one girder of length L (Fig. 1) is considered. Such frames are typical for many technical systems, for example industrial facilities or portal cranes. The columns are fixed in the foundation and the joints between columns and girder are rigid. The frame is loaded by a vertical crane force F_V which can be located arbitrarily between two limit positions and is always directed downwards. The operational field of this force is marked gray in Fig. 1. In addition, the crane truck can cause a horizontal brake force F_B at the same position as F_V. The direction of F_B could be either to the left or to the right. Finally, the left column is loaded horizontally by a force F_H acting always from the left to the right. The position of F_H is fixed. All loads are considered as static ones and, thus, the problem to be solved is a static problem.

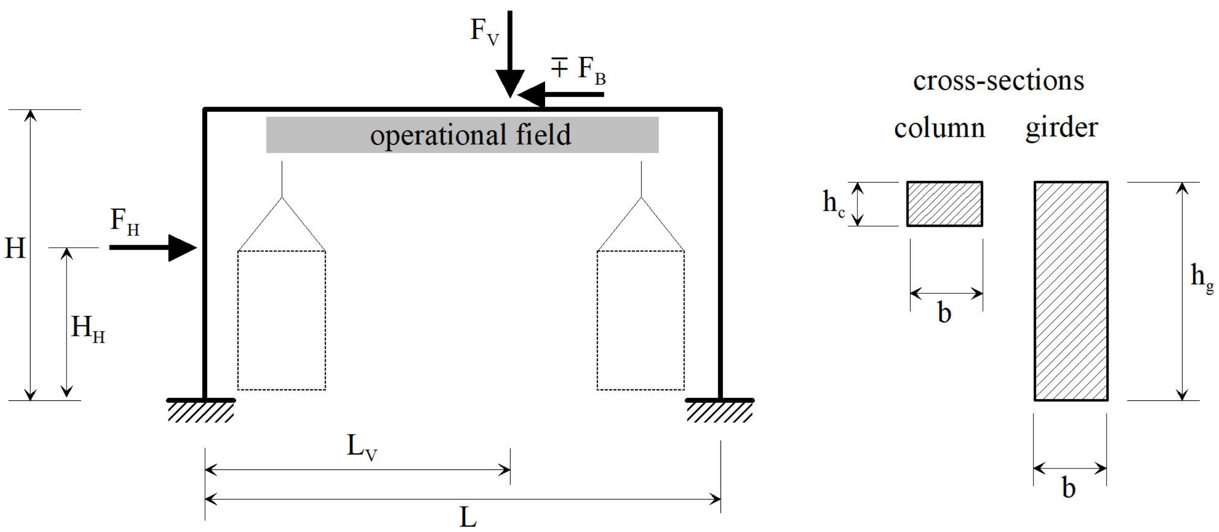

Figure 1: Mechanical model of the portal frame

The columns and the girder are made of steel and have rectangular cross-sections of the same width and different heights, as shown in Fig. 1. Real hollow steel profiles are not considered here for simplicity, since the cross-sections cannot be varied continuously but taken from an assortment list at disposal. The material behavior is assumed to be linear elastic until failure.

2.2 Potential failure modes

It is assumed that the frame suffers only deformations in the plane, i.e. no out-of-plane deformations are considered. Under given loads, the frame experiences both normal forces N, lateral forces Q and bending moments M. Typical distributions of these internal forces as well as a typical deformation state are given exemplarily in Fig. 2 for better understanding of system behavior. Two failure modes with respect to ultimate limit states are considered: material failure and stability failure.

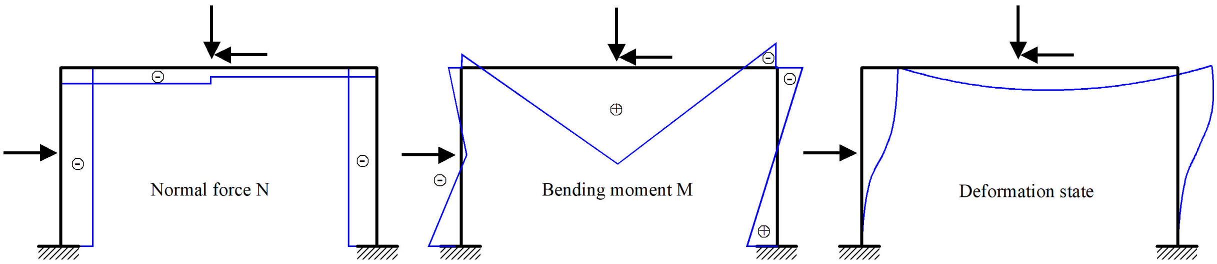

Figure 2: Internal forces N and M as well as deformation state of the portal frame

Material failure (local stress problem) The material failure is always local and occurs due to the violation of a limit stress, in this case, the yielding stress of steel σ_{y}. The maximum acting stress can be calculated from the known normal forces and bending moments in each cross-section as follows:

$$\sigma_{\max} = \frac{N}{A} \pm \frac{M}{W} \tag{1}$$

with the cross-section area A and the resistance moment W of a rectangular cross-section according to Fig. 1

$$A = bh \quad \text{and} \quad W = \frac{bh^2}{6}\,. \tag{2}$$

The limit state function for material failure is implicit since it requires each time the calculation of internal forces and corresponding maximum stresses at critical locations:

$$g\left(\sigma_{\max}, \sigma_{\mathrm{y}}\right) = \frac{\sigma_{\mathrm{y}}}{|\sigma_{\max}|} - 1.0 = \frac{\lambda_{\mathrm{mat}}}{\lambda} - 1.0 = 0. \tag{3}$$

Here, λ_{mat} and λ indicate the critical load factor for material failure and the current load factor $\lambda = 1.0$, respectively.

Stability failure (global buckling) The stability failure occurs due to the loss of the system equilibrium and is, therefore, always global. Stability failure is typically characterized by the buckling load $\lambda_{\mathrm{stab}}P$ and the buckling mode Φ, i.e. the deformation state appearing after buckling. At that, the load factor λ_{stab} marks the critical value of a given load case P. A typical buckling mode of the portal frame under consideration is shown in Fig. 3.

The corresponding limit state function is also implicit since it requires a step of the system stability analysis and can be written in terms of the buckling load $\lambda_{\mathrm{stab}}P$ and a given load λP as follows:

$$g\left(\lambda, \lambda_{\mathrm{stab}}\right) = \frac{\lambda_{\mathrm{stab}}}{\lambda} - 1.0 = 0\,. \tag{4}$$

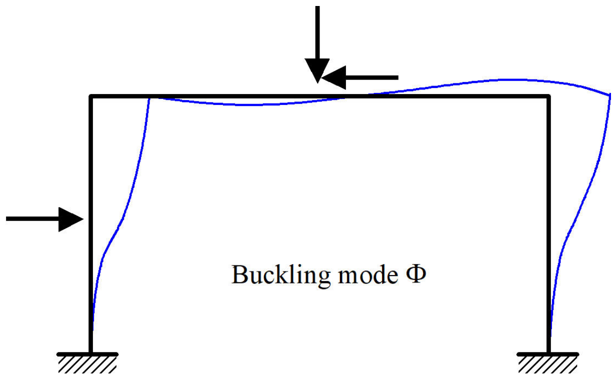

Figure 3: Buckling mode of the portal frame

System failure (material or stability failure) It is assumed that any local exceedance of yielding stress and any buckling of the frame are unacceptable limit states for the crane operation and, thus, correspond to the system failure. The load, structural and material parameters in this benchmark are chosen in such a way that both material and stability failure occur at similar load levels. Therefore, the real failure mode is sensitive to uncertainties.

2.3 Structural analysis

Structural analysis of the frame is carried out by the displacement method similarly to the finite element method. All necessary explanations are given below and provided in the appendix to the benchmark, so that all participants can use the same computational model. Thus, the model uncertainty can be excluded from this test aiming at the role of polymorphic data uncertainties.

The stiffness relation of the Bernoulli beam elements with the stiffness matrix K_e, load vector P_e and the nodal displacement vector v_e is well known [3]:

$$K_\mathrm{e} \cdot v_\mathrm{e} = \begin{bmatrix} \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ & \frac{12EI}{l^3} & -\frac{6EI}{l^2} & 0 & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ & & \frac{4EI}{l} & 0 & \frac{6EI}{l^2} & \frac{2EI}{l} \\ & & & \frac{EA}{l} & 0 & 0 \\ & & & & \frac{12EI}{l^3} & \frac{6EI}{l^2} \\ \text{symm.} & & & & & \frac{4EI}{l} \end{bmatrix} \cdot \begin{bmatrix} u_\mathrm{l} \\ w_\mathrm{l} \\ \varphi_\mathrm{l} \\ u_\mathrm{r} \\ w_\mathrm{r} \\ \varphi_\mathrm{r} \end{bmatrix} = \begin{bmatrix} N_\mathrm{l} \\ Q_\mathrm{l} \\ M_\mathrm{l} \\ N_\mathrm{r} \\ Q_\mathrm{r} \\ M_\mathrm{r} \end{bmatrix} = P_\mathrm{e}. \tag{5}$$

It provides exact solution with respect to displacements:

$$v_\mathrm{e} = K_\mathrm{e}^{-1} \cdot P_\mathrm{e}. \tag{6}$$

The portal frame is discretized by five classical Bernoulli beam elements and 12 active degrees-of-freedom (nodal displacements), as shown in Fig. 4.

Its linear response to static loads can be calculated by use of the system stiffness relation, i.e. system of algebraic equations:

$$K \cdot V = \lambda P \quad \text{with} \quad \lambda = 1. \tag{7}$$

The elastic system stiffness matrix $K_{ij}, i, j = 1, \ldots, 12$ and the system load vector $P_i, i = 1, \ldots, 12$ are derived analytically according to the system discretization in Fig. 4 and stay explicitly for all participants at disposal. A MATLAB® file executing an exemplary deterministic calculation is available under [15] in order to minimize mistakes and misunderstanding with respect to the structural analysis.

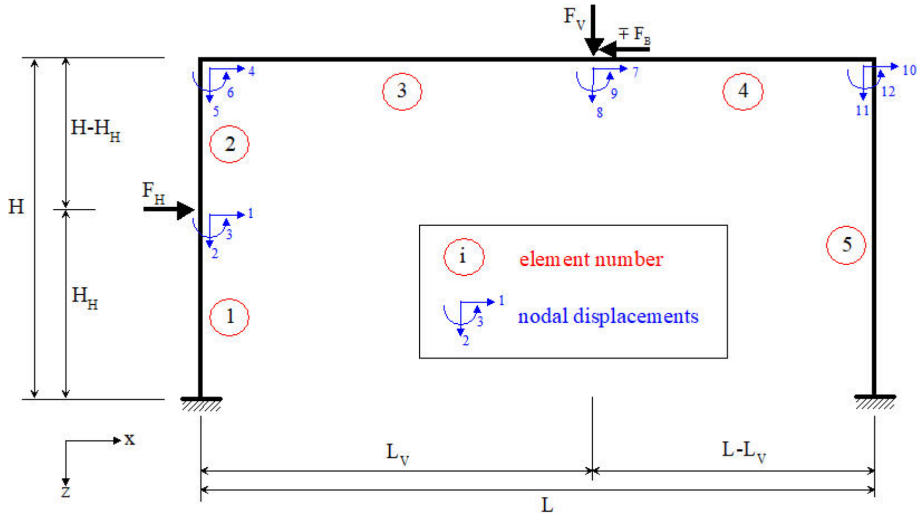

Figure 4: Discretized mechanical model with five beam elements and 12 nodal degrees-of-freedom

At that, the following input parameters are defined: material properties E, σ_{y}; geometry L, H; element cross-sections $b_i, h_i, i = 1, \ldots, 5$; load magnitudes F_{V}, F_{B}, F_{H} and load positions L_{V}, H_{H}.

Material failure (local stress problem) The analysis steps for evaluating the limit state function according to Eq. (3) are shortly described in the following:

1. calculate the system nodal displacements: $V = K^{-1} \cdot P$
2. transform the system displacements V to the local element ones $v_{\mathrm{e},i}$
3. calculate the internal forces of each element: $s_{\mathrm{e},i}$ according to Eq. (5)
4. calculate the maximum stress in Eq. (1) of each element: $\sigma_{\max,i}$
5. calculate the maximum stress in portal frame: $\sigma_{\max} = \max(\sigma_{\max,i})$
6. calculate the critical load factor $\lambda_{\mathrm{mat}} = \frac{\sigma_y}{|\sigma_{\max}|}$
7. calculate the limit state function value according to Eq. (3): $g\left(\sigma_{\max}, \sigma_{\mathrm{y}}\right)$

If the frame losses its stability and buckles, it takes a new deformed state called buckling mode. The critical load at which it happens is called buckling load. The buckling load factor λ and the buckling mode Φ can be calculated according to the classical stability theory from the generalized eigenvalue problem for the elastic (K) and the geometric (K_{g}) system stiffness matrices as follows:

$$(K - \lambda K_{\mathrm{g}})\Phi = 0 \qquad (8)$$

The geometric system stiffness matrix K_{g} of the frame accounts for the deformed state and depends on the acting normal forces N_i and, thus, on the load level λP. The linearized geometric stiffness matrix $K_{\mathrm{g,e}}$ for classical beam elements is well known [3]. The geometric system stiffness matrix K_{g} of the frame with respect to 12 active nodal degrees-of-freedom (Fig. 4) is derived analytically by the authors from the element matrices $K_{\mathrm{g,e}}$ and stays at disposal for all participants. This kind of stability analysis provides only a linearized prediction of the buckling load and does not account for imperfections. However, it is considered as sufficient for the purpose of the present benchmark.

Stability failure (global buckling) The analysis steps for evaluating the stability limit state function according to Eq. (4) are shortly described in the following:

1. calculate normal forces from the linear elastic analysis at $\lambda = 1.0$: N_i
2. calculate the geometric system stiffness matrix: K_g
3. solve the eigenvalue problem in Eq. (8) and determine the minimum eigenvalue, i.e. load factor: $\lambda_{stab} = \min(\lambda_i)$
4. calculate the limit state function $g(\lambda, \lambda_{stab})$ according to Eq. (4).

2.4 Result interpretation

Material failure occurs if $g(\sigma_{max}, \sigma_y) \leq 0$, stability failure takes place if $g(\lambda, \lambda_{stab}) \leq 0$. The failure mode with the smallest load factor λ_{stab} or λ_{mat} occurs first.

3 Available data

3.1 Input data

The geometry, load, material and cross-sectional parameters are defined with uncertainties as described below.

Geometry and cross-sections The lengths of the columns and the girder are explicitly measured and considered as deterministic: $L = 10$m and $H = 8$m (see Fig. 1). The quality management of the manufacturer gives a usual fabrication tolerance of $\pm$2mm for the cross-sections of the frame. According to the original design, the cross-sections of the members should be as follows:

- width: $b_1 = b_2 = b_3 = b_4 = b_5 = 0.20$m
- height: $h_1 = h_2 = h_5 = 0.14$m (column), $h_3 = h_4 = 0.85$m (girder).

Loads The operation field of the vertical load F_V is defined deterministically within two limit positions from the left edge $L_V \in [2.0\text{m}, 8.0\text{m}]$ (Fig. 1). The position L_V of the vertical load within this interval can be arbitrary. It is known that usual live loads of the crane vary between 1000kN and 2000kN. A special sensor prevents the crane operation if the vertical load is larger than 3000kN.

The crane also causes brake loads when moving freight. Depending on the movement velocity and type of braking, the brake force can be determined with regard to the vertical force as follows:

$$|F_B| = \alpha F_V \quad \text{with} \quad \alpha \in [0.0001, 0.001] \; . \tag{9}$$

The brake force F_B can be directed both to left or right, while the vertical load F_V is directed always downwards.

The horizontal load F_H results from the wind and attacks the frame always at the fixed position $H_H = 4.0$m from left to right. During two separate measurement campaigns, each of five months duration, extreme values of the wind load F_H per month have been measured, see Table 1. The dead load of the portal frame is ignored for simplicity.

Table 1: Measured extreme loads F_{H}

Measurement 1	7.0kN	2.8kN	5.8kN	8.3kN	10.3kN
Measurement 2	10.1kN	4.6kN	12.4kN	8.2kN	15.7kN

Material It is known that the frame is built of steel S 355 [7] with the characteristic value of the yield strength of 275MPa, that is the 5%-quantile value of the statistical distribution. The mean value of the Young's modulus E can be taken as 210000MPa. According to the Probabilistic Model Code [12], log-normal distributions are recommended to be applied with a coefficient of variation of 7% for the yield strength σ_{y} and with a coefficient of variation of 3% for the Young's modulus E.

4 Challenges

4.1 Challenge 1: Decision making

The engineering decision to be taken sounds: Is the operation of the frame under conditions described above allowed or not? The participants are expected to give the answer "yes" or "no". The answer "yes" is correct, if the failure rate remains smaller than 10^{-3}, i.e. 1 failure per 10^3 load operations, that is the requirement of the operator. The answer "no" is correct, if the failure rate is larger than 10^{-3}. Besides the answer "yes" or "no", participants are expected to explain their classification of uncertainties, suitable approaches to handle them and the background of the decision. An independent check of the decision taken by every participant can be done by uncertain simulations of the frame with "true" structural and load parameters, which are known to the authors. The true value of the parameters and the resulting failure rate will be declared and explained.

In a second step, the decision can be improved by using $N = 5000$ data sets for the load parameters F_{H}, F_{B} and F_{V}. These are given in a text file under [15].

4.2 Challenge 2: Decision making by data assimilation

The engineering decision to be taken sounds: Is the operation of the frame under conditions described above allowed, if the measurement data available from the operation is taken into account? The participants are expected to give the answer "yes" or "no". The answer "yes" is correct, if the failure rate remains smaller than 10^{-3}, i.e. 1 failure per 10^3 load operation. The answer "no" is correct, if the failure rate is larger than 10^{-3}. The focus of this challenge is directed toward the data assimilation and its contribution to the uncertainty quantification. The true material and cross-sectional parameters are expected to be identified and specified. Again, an independent check of the decision will be done, and the true values of the parameters and the resulting failure rate will be declared and explained.

Operation measurement data During operation, that means without occurrence of failure, the response parameters given in Table 2 have been measured in addition to the associated load parameters F_{H}, F_{B} and F_{V}, see the text file under [15]. These data sets have been generated numerically, by calculating the frame with the true structural parameters. Several measurement errors and measurement noise are already incorporated into the data sets.

Remarks to the measurement data Due to the stiffness relations in the frame, it can be assumed that the whole girder experiences the same horizontal displacement. The vertical displacement V_8 is measured at the measured load position L_{V}, so both values belong together in each measurement. All measurements have been performed by a laser distance meter, which provides a nominal accuracy of ±1mm. It is also known that some measurements could be disturbed

Table 2: Measurements during operation

Measured value	**Denomination**
horizontal displacement [m]	V_4
vertical displacement [m]	V_8
position of the vertical load [m]	L_V

by foreign objects breaking the laser rays during measurements. In this case, measurements contain wrong values.

4.3 Challenge 3: Design under uncertainties

The final challenge is to design the frame in such a way that the failure rate becomes smaller than 10^{-4}. The only design parameters are the cross-section heights of the two columns $h_1 = h_2 = h_5$ and that of the girder $h_3 = h_4$. All other parameters are the same as in challenge 1. The best design is that one with the minimum weight of the frame and the target failure rate fulfilled. An independent check of each design can be done individually by uncertain simulations of the frame with "true" structural and load parameters and the design values of the cross-section height provided by each participant.

5 Final remarks

Researchers dealing with uncertainty quantification are invited to participate in this benchmark study and submit their solutions to the corresponding author via e-mail (yuriy.petryna@tu-berlin.de). Each solution will be registered and independently checked as described above. A joint publication including comparison of all submitted results will be prepared and discussed with the participants.

The information on the benchmark as well as all necessary data are available under [15].

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