Biomed Tech 2013; 58 (Suppl. 1) © 2013 by Walter de Gruyter · Berlin · Boston. DOI 10.1515/bmt-2013-4423

CONTROL OF A MOBILE REHABILITATION ROBOT USING EXACT FEEDBACK LINEARISATION

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Abstract: *This contribution is concerned with the feedback control of a table-placed mobile rehabilitation robot using exact feedback linearisation to precisely track arbitrary position and orientation profiles. An outer control loop exactly linearises and decouples the nonlinear kinematic robot model. This loop also generates reference velocities for the three omnidirectional wheels of the robot that are feedback controlled by individual digital controllers on a inner loop level. The concept was validated in simulations.*

Keywords: *Nonlinear Control, Robotics, Rehabilitation*

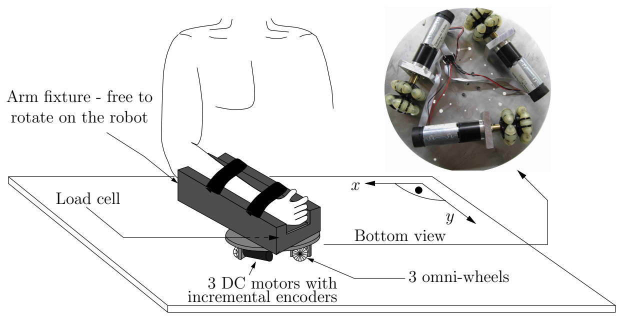

Figure 1: An application scenario of the *Reha-Maus*.

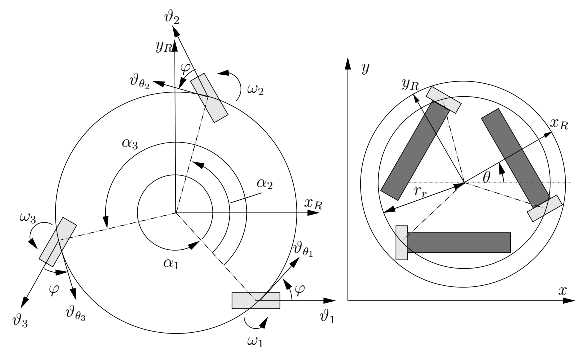

Figure 2: *Reha-Maus*: Geometry and coordinate systems.

Introduction

Robot-aided neuro-rehabilitation has been widely studied in recent years. A variety of upper limb rehabilitation robots has been developed to assist, enhance, evaluate, and document neurological and orthopaedic rehabilitation of movement. However, these devices exclusively focused on the clinical setting which entails a lack of mobility, high acquisition costs and limited patient training times.

The *Reha-Maus*, which is a novel upper limb rehabilitation system developed by the Control Systems Group at TU-Berlin, represents one of the first concepts of a portable rehabilitation robot that actively provides different levels of patient assistance [1]. The design of the *Reha-Maus* is based on a mobile robot driven by omni-directional wheels. This enables rotational and translation motion in a plane for guiding the hand/lower arm.

However, the previously realised control scheme only allowed the arbitrary tracking of reference positions while the orientation had to be kept nearly constant [1]. Unwanted changes in the orientation, e.g. by external disturbances, could even render the position control loop unstable.

This contribution describes a novel motion control system for the *Reha-Maus* that involves nonlinear control theory in order to enable the generation of arbitrary position and orientation profiles.

Methods

Omni-directional robot and kinematic model

The *Reha-Maus* is designed to allow patients to train their hemiparetic arm. Figure 1 shows a prospective application scenario of the *Reha-Maus*. The lower right arm of a patient is pivoted on the robotic platform. The system is used to assist or resist the patient's arm and shoulder movements during the training. Human-device interaction forces/moments can be measured by a 6D force/torque sensor underneath the arm support, and the movement is monitored by an infrared camera above the table and incremental encoders at the motors. Arbitrary translational and rotational motion on the table surface is facilitated by three DC-motor-driven omni-wheels. More technical details are given in [1].

The *Reha-Maus* possesses three DoFs and has a fixed body frame $[x_R, y_R]$, aligned to the centre of mass (cf. Figure 2). The description of the kinematics and dynamics takes place in generalised coordinates $\boldsymbol{q} = [x, y, \theta]^T$, where x and y represent the position of the robot on the planar workspace and θ is the robot orientation. A kinematic model was derived in [1] based on the assumption that the wheels have no slippage in the direction of traction force. The angular velocities of the three omni-wheels are forming the vector $\boldsymbol{\omega}(t) = [\omega_1(t), \omega_2(t), \omega_3(t)]^T$. The kinematic relation between the wheels' angular velocity vector $\boldsymbol{\omega}(t)$ and the gen-

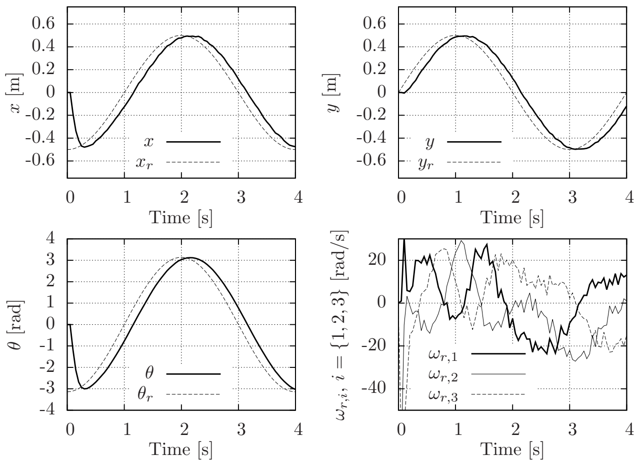

Figure 3: Results for tracking a circular trajectory with simultaneous robot rotation.

eralised velocity vector $\dot{\boldsymbol{q}}(t)$ is defined as

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{\Gamma}(\theta(t))\boldsymbol{\omega}(t) \quad (1)$$

$$\boldsymbol{\Gamma}(\theta(t)) = r_\omega \begin{pmatrix} \cos(\theta) & \cos(\theta + \frac{2\pi}{3}) & \cos(\theta + \frac{4\pi}{3}) \\ \sin(\theta) & \sin(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{4\pi}{3}) \\ \frac{\sin(\varphi)}{r_r} & \frac{\sin(\varphi)}{r_r} & \frac{\sin(\varphi)}{r_r} \end{pmatrix},$$

where $r_r = 10.5$ cm is the robot rotation radius, and $\varphi = 39°$ is a construction specific angle. The scalar r_ω is the radius of one omni-wheel. A discrete-time kinematic model can be obtained by the Euler method with the sampling period $t_s = 50\,\text{ms}$:

$$\boldsymbol{q}(k+1) = \boldsymbol{q}(k) + \overline{\boldsymbol{\Gamma}}(\theta(k))\boldsymbol{\omega}(k). \quad (2)$$

Here, k is the sample index and $\overline{\boldsymbol{\Gamma}}(\theta(k)) = t_s\boldsymbol{\Gamma}(\theta(k))$.

Control system

To perform arbitrary robot movements, a cascaded discrete-time control structure is applied.

An inner loop consists of three independent motor speed controllers that operate the system at maximum actuator capacity in order to facilitate the simplified state-space description of the robot given below. The sample rate of the inner loop is 1 kHz and a bandwidth of approximately 45 Hz is achieved. The design of each motor speed controller is based on an experimentally identified transfer-function model and is described in [1].

An outer loop regulates the generalised robot coordinates and generates as control signals the reference wheel velocities for the inner loop. The sampling time of the outer loop is set to 50 ms. The relationship between the wheel velocity reference vector $\boldsymbol{\omega}_r$ and controlled wheel velocity vector $\boldsymbol{\omega}$ can be expressed as a simple time delay of one sampling step:

$$\boldsymbol{\omega}(k+1) = \boldsymbol{\omega}_r(k). \quad (3)$$

Combining the kinematic model (2) with the controlled wheel speed dynamics (3) yields a non-linear model with the state $\boldsymbol{x}(k) = [\boldsymbol{q}(k)^T, \boldsymbol{\omega}(k)^T]^T$, the output $\boldsymbol{y}(k) = \boldsymbol{q}(k)$ and the input signal $\boldsymbol{\omega}_r(k)$. This model is employed for designing the outer loop controller. The entire state $\boldsymbol{x}(k)$ is accessible by measurement or estimation [1]. In order to exactly linearise and to decouple the discrete-time multivariable non-linear model, the following relation between the output $\boldsymbol{y}$ and the input $\boldsymbol{\omega}_r$ is derived from the state-space model (Eqs. (2) and (3)):

$$\boldsymbol{q}(k) = \boldsymbol{q}(k-1) + \overline{\boldsymbol{\Gamma}}(k-1)\boldsymbol{\omega}_r(k-2). \quad (4)$$

The control law

$$\boldsymbol{\omega}_r(k) = \overline{\boldsymbol{\Gamma}}^{-1}(k+1)(\boldsymbol{v}(k) - \boldsymbol{q}(k+1))$$

leads to $\boldsymbol{q}(k) = \boldsymbol{v}(k-2)$ with the new input signal $\boldsymbol{v}$. The now linearised and decoupled plant model represents a simple time delay of two sampling steps with respect to signal $\boldsymbol{v}$. The term $\boldsymbol{q}(k+1)$ can be calculated from $\boldsymbol{q}(k)$ and $\boldsymbol{\omega}(k)$ using the state propagation described in Eq. (2). For each generalised coordinate, the exactly linearised plant will be separately feedback controlled by a second order standard linear digital controller with integral action [2] to allow the tracking of a given reference (x_r, y_r, θ_r). The poles of the outer closed-loop dynamics have been chosen to obtain a bandwidth of approximately 2 Hz.

Results

The cascaded control scheme with the nonlinear controller at the outer loop was evaluated in simulations first. Figure 3 shows the results of a tracking test. The robot had to follow a circular path within 4 s while rotating at the same time around its axis. Noise, typically observed at the real system, was added to the states during simulation. The robot was initially in the centre of the circle with an orientation error of 180°.

Discussion

The proposed nonlinear control scheme successfully linearises and decouples the nonlinear discrete-time model and therefore allows the tracking of arbitrary position and orientation profiles. After the compensation of initial errors, the system output follows the references as specified. The observed wheel velocities are feasible in practice. The developed controller forms the basis for the realisation of many therapeutic exercises that will be realised by the robot. The experimental validation of the control concept is ongoing.

Bibliography

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